MTH 201.4 SS 2007 EXAM 3 **Name:** ANSWER KEY **PID:** A00000000

Instructions:

You must show ALL your work in ALL questions. You will be graded on your methods, not just your answers. Use only the space provided for each question. Any usage of calculators is prohibited during the exam.

You will have EXACTLY 60 minutes for the exam, which consists of problems numbered 1 – 11. Request a new copy of the exam if any of the problems are missing or hard to read.

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1) *(2 points each)* Fill in the blanks using the correct terminology in their full form:

a) The fraction rule for multiplication states that:
For each pair of fractions $\frac{a}{b}$ & $\frac{c}{d}$ $(b,d \neq 0)$ ① $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ holds. ① (in Handout).

b) The mixed number $3\frac{2}{7}$ is equal to the improper fraction $\frac{23}{7}$ ②. (EMT p138 etc)

c) A factory has 100 workers. There are 40 men and the rest are women.
How many percent more women than men are there?

In the above problem, the whole unit is the number of men ②. (EMT p178 - Table)

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2) *(2 points)* Complete the following definition: (Hint: The space provided is sufficient!)

A *proportion* is a statement that 2 ratios are equivalent ② (EMT p169).

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3) *(2 points each)* True or False:

a) Ratios can be subtracted in a consistent manner. T (F) (EMT p172)

b) The numerator of a fraction represents the number of fractional units. (T) F (EMT p132)

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4) *(2 points each)* Show how the following can be calculated *mentally*, without converting to fractions.

a) Ten *times* "two and one fifth": $10 \times (2 + \text{"one fifth"}) = (10 \times 2) + (10 \times \text{"one fifth"}) = 20 + \text{"10 fifths"} = 20 + 2 = 22$ DP ②

b) "Six fifths" *divided by* three: "6 fifths" $\div 3 = (6 \div 3)$ fifths $= 2$ fifths ②

(EMT p135 & p144)

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5) *(3 points each)* Show (*clearly* but *briefly*) how to compute the following using Mental Math. Write down the *intermediate steps*, and state the *properties/techniques* used.

a) $36\times 9\frac{7}{12} = 36\times\left(10-\frac{5}{12}\right) = (36\times 10)-\left(3\cancel{6}\times\frac{5}{\cancel{12}}\right) = 360-15 = 345$

PV ① DP ① PV PV ①

[HW Set 26 Problem 2c (simplified)]

b) $2\cdot\frac{7}{11}+2\cdot\frac{4}{11} = 2\left(\frac{7}{11}+\frac{4}{11}\right) = 2\cdot\frac{11}{11} = 2\cdot 1 = 2$

DP ② ①

[HW Set 26 Problem 2a (simplified)]

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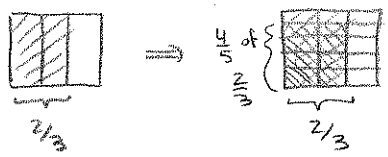
6) *(5 points)* Illustrate the multiplication $\frac{4}{5}\times\frac{2}{3}$ using an area model in two steps.

$\frac{4}{5}\times\frac{2}{3} = \frac{4}{5}$ of $\frac{2}{3}$ ①



① ②

8 double-shaded tiles out of 15, i.e. $\frac{8}{15}$

So: $\frac{4}{5}\times\frac{2}{3} = \frac{8}{15}$ ①

[Study Guide, #46] [EMT p145]

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7) *(8 points)* For two fractions $\frac{a}{b}$ and $\frac{c}{d}$ with $b,d\neq 0$, $\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$.

Prove the above statement using the arithmetic properties and fraction rules, stating them when they are used in the steps.

$\frac{a}{b}\cdot\frac{c}{d} = \frac{a\cdot c}{b\cdot d} = \frac{ac}{bd}$ ④

R4 ④

[Study Guide, #5 (simplified)] [EMT p160]

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8) *(10 points)* Use fraction arithmetic to simplify. State the rules or properties used at each step.

$\frac{3}{8}\cdot\left(\left(\frac{5}{6}-\frac{1}{2}\right)\div\frac{5}{2}\right) = \frac{3}{8}\cdot\left(\frac{5}{6}-\frac{1}{2}\right)\cdot\frac{2}{5}$ (R5') ②

$= \frac{3}{8}\cdot\left(\frac{5}{6}-\frac{3}{6}\right)\cdot\frac{2}{5}$ (R1) ②

$= \frac{3}{8}\cdot\frac{2}{6}\cdot\frac{2}{5}$ (R2) ②

$= \frac{\cancel{3}\cdot 2\cdot 2}{48\cdot\cancel{5}\cdot 5}$ (R4) ②

$= \frac{1}{20}$ (R1) ②

[HW Set 29 Problem 1b]

[Also: HW Eval. 15 Problem 1b – Solved in its key]

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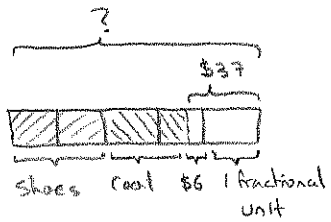
9) *(10 points each)* Give full Teacher Solutions for the following problems.

a) (Use bar diagrams) → STUDY GUIDE: Type 1

Tony spent $\frac{2}{5}$ of his money on a pair of shoes. He also bought a coat which cost \$6 less than the shoes. He had \$37 left. How much money did he have at first?



④

1 fractional unit = 37 − 6 = 31 ③

Whole: 5 fr. units = 5 × 31 = 155

He had \$155 at first. ③

[HW27 Problem 3c]

b) (Use algebra) → STUDY GUIDE: Type 2

Sally and Susan had the same amount of money initially. After Sally spent \$15 and Susan spent \$24, the ratio of Sally's money to Susan's was 4 : 3. How much money did each girl have at first?

Let A & B denote Sally's & Susans final amounts of money, respectively. ②

Final ratio: A : B = 4 : 3 ⟹ A = 4x & B = 3x for some x. ②

Initial amounts: $4x + 15 = 3x + 24$

$4x - 3x = 24 - 15$

$x = 9$ ②

So: Initial amount $= 4x + 15$

$= 4(9) + 15$

$= 36 + 15 = 51$ ②

They both had \$51 at first. ②

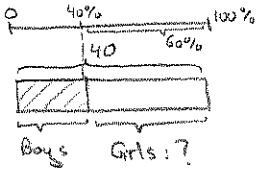
[HW31 Problem 1 (PMGA p38 #4)]

[Also see HW Bral 16 Key for alternate method!]

c) (Use the unitary method) → STUDY GUIDE: Type 3

In a class of 40 students, 40% are boys. 50% of the girls wear glasses. How many girls wear glasses?

Step 1:



②

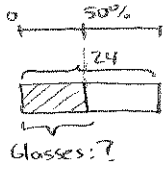
#	%
40	100%
4	10%
24	60%

(÷10, ×6)

②

There are 24 girls.

Step 2:



②

#	%
24	100%
12	50%

(÷2)

②

12 girls wear glasses. ②

[HW32 Problem 1 (PMGA p54 #10) SIMPLIFIED!]

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10) Consider the partitive interpretation (PD) of the division $\frac{2}{3} \div \frac{7}{5}$.

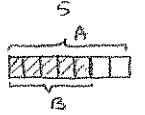
a) *(3 points)* What is the interpretive question? "$\frac{2}{3}$ is $\frac{7}{5}$ of what?" [EMT p146]

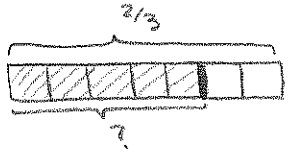
b) *(4 points)* Make up a short word problem for this division and interpretation.

Problem 1: It takes $\frac{2}{3}$ of a minute to fill $\frac{7}{5}$ gallons of water. How long would it take to fill 1 gallon?

Problem 2: It takes $\frac{7}{5}$ of a minute to fill $\frac{2}{3}$ gallons of water. How many gallons would fill in 1 minute?

c) *(4 points)* Draw the bar diagram and find the solution using the bar diagram.

(Intermediate step: A is $\frac{7}{5}$ of B: )



② 7 fractional units = $\frac{2}{3}$

1 fractional unit = $\frac{2}{3} \div 7 = \frac{2}{21}$

Whole = 5 fractional units = $5 \times \frac{2}{21} = \frac{10}{21}$ ②

[See: EMT p148, HW set 26 Problem 11, HW set 27 Problem 2, HW set 28 Problems 4 & 8]

d) *(4 points)* Show how the diagram and the solution lead to the "invert and multiply" rule.

$\frac{2}{3} \div \frac{7}{5} = \left(\frac{2}{3} \div 7\right) \times 5 = \frac{2}{3} \times (5 \div 7) = \frac{2}{3} \times \frac{5}{7}$

↑ as seen from the diagram & the solution; ↑ A10; ↑ R3

so we got "invert & multiply"

[Each step: ①]

[Also: Study Guide Items: 4c, d]

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11) *(10 points)* For two fractions $\frac{a}{b}$ and $\frac{c}{b}$ with the same denominator $b \neq 0$, $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$.

Show how the above rule follows only from the Fractional Unit Property and the Distributive Property, stating them when they are used in the steps.

$\frac{a}{b} + \frac{c}{b} = \underbrace{\left(a \cdot \frac{1}{b}\right) + \left(c \cdot \frac{1}{b}\right)}_{①}$ (FUP) ②

$= \underbrace{(a+c)\left(\frac{1}{b}\right)}_{②}$ (D.P.) ②

$= \underbrace{\frac{a+c}{b}}_{①}$ (FUP) ①

[Study Guide Item 5a-(i) Also shown in its entirety in Handout]

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