MTH 201.4 SS 2007 EXAM 2 **Name:** ANSWER KEY **PID:** A00000000

Instructions:

You must show ALL your work in ALL questions. You will be graded on your methods, not just your answers. Use only the space provided for each question. Any usage of calculators is prohibited during the exam.

You will have EXACTLY 50 minutes for the exam, which consists of problems numbered 1 – 14. Request a new copy of the exam if any of the problems are missing or hard to read.

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1) *(2 points each)* Fill in the blanks:

a) Jane put 14 pens equally into 4 boxes. When she was done;
There were 3 pens in each box, a total of 12 pens distributed, and 2 left over.

b) After each step in long division, it's necessary to check that $0 \leq$ remainder < divisor.

c) The most important property used in proving $(a + b)(c + d) = ac + bc + ad + bd$ is:
the distributive property of multiplication over addition.

d) There are infinitely many prime numbers.

e) In the addition 107 + 235, the ones denomination is rebundled.
i.e. "composed"

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2) *(2 points each)* Complete the following definitions: (Hint: The space provided is sufficient!)

a) An *algorithm* is a systematic step-by-step procedure to solve a class of problems.

b) An *equation* is a statement that two expressions are equal.

c) A is *divisible* by k if A = $k \cdot a$ for some whole number a..

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3) *(2 points each)* True or False:

a) 5 + 13 ÷ 8 is an algebraic expression. T F → any numerical expression is also an algebraic expression

b) 321 is a prime number. T F → $321 = 3 \cdot 107$

c) All prime numbers are odd. T F → 2 is the only even prime.

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4) *(2 points each)* Calculate the following using mental math techniques that involve,
either squares of numbers up to 20
or powers of 2 up to the tenth power
or arithmetic identities including squares of sums/differences, differences of squares.
Clearly, but briefly, indicate the steps involved.

a) $16 \times 14 =$ $(15+1)(15-1) = 15^2 - 1^2 = 225 - 1 = 224$
① diff. of squares ①

b) $16 \times 32 =$ $2^4 \times 2^5 = 2^9 = 512$
① ①

c) $49^2 =$ $(50-1)^2 = 50^2 - 2(50)(1) + 1^2 = 2500 - 100 + 1 = 2401$
① square of difference ①

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5) *(2 points)* Explain, in a _single short sentence_, ONLY the mistake made:

$$\begin{array}{r} 25 \\ +\ 89 \\ \hline 1014 \end{array}$$

The 10 bundles of ten are recorded in wrong (shifted) denominations.

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6) *(5 points)* Circle the numbers below that divide 243,276.

2 3 4 ~~5~~ ~~8~~ ~~9~~ ~~10~~ 11

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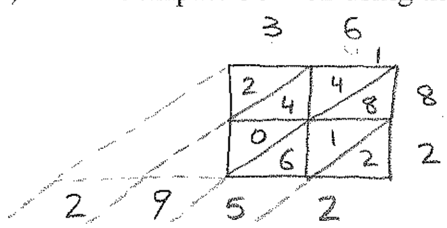
7) *(5 points)* Compute 36×82 using the lattice method.



Correct lattice: ①
Correct numbers: ②
Correct result: ②

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8) *(5 points)* _Complete_ the following long division:

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       1         → ①
       5 0       → ①
43 | 2 5 9 4
   - 2 1 5       → ①
       4 5  ]
     - 4 4  ]    → ①
       1 4
```

SO: $2594 \div 43 = 60$ R14 → ①

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9) *(6 points)* Find the prime factorization of 1495 using its factor tree (and primality tests).

$1495 \div 5 = 2990 \div 10 = 299$ ①

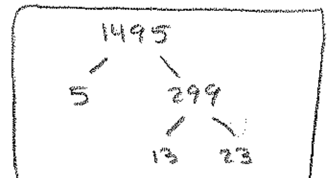
$289 < 299 < 324$
$17 < \sqrt{299} < 18$ ①

Need to check:
2, 3, 5, 7, 11, 13, 17 ①

Last digit: 9 → ~~2~~, ~~5~~
Sum of digits: 20 → ~~3~~
Alt. sum: 2 → ~~11~~ ①

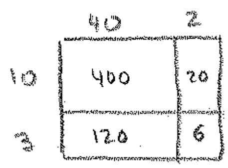
7: $299 = 280 + 19$ ✓ ✗

13: $299 = 260 + 39$ ✓ ✓ ⟹ $299 = 13 \times (20+3)$ ①



SO: $1495 = 5 \cdot 13 \cdot 23$ ①

10) *(5 points)* Illustrate the use of the distributive property in the multiplication algorithm of 42×13 using a labeled rectangular array, showing the values appearing in the algorithm.

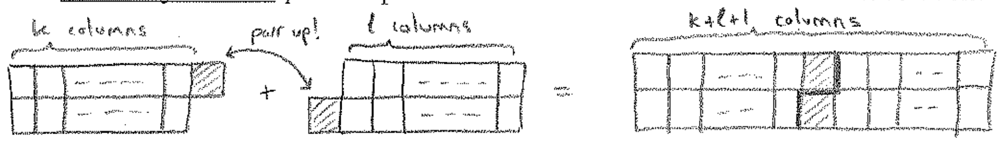


lengths: ②
areas: ②

SO:

$$\begin{array}{r} 42 \\ \times\ 13 \\ \hline 6 \\ 120 \\ 20 \\ 400 \\ +\quad \\ \hline 546 \end{array}$$ ②

11) *(6 points)* Give a correctly labeled picture proof that the sum of two odd numbers is even.



first odd #: $m = 2k+1$ tiles

second odd #: $n = 2l+1$ tiles

sum: $m+n = 2(k+l+1)$ tiles, so even!

□

each diagram: ① each labelling: ①

12) *(8 points)* Give an algebraic proof of the identity $(a + b)(a - b) = a^2 - b^2$.

$$\begin{aligned} (a+b)(a-b) &= a(a-b) + b(a-b) && \text{DP} \\ &= a^2 - ab + ba - b^2 && \text{DP} \\ &= a^2 - \cancel{ab} + \cancel{ab} - b^2 && \text{CP} \\ &= a^2 - b^2 && \text{DP} \end{aligned}$$

each step: ②

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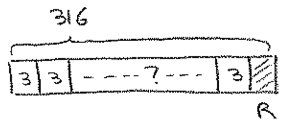
13) *(10 points each)* Give full Teacher Solutions for the following problems.

a) (Use a diagram and long division)
A man has 316 oranges. He puts 3 oranges in each bag.
How many bags of oranges can he make? How many oranges will be left over?



③

$$\begin{array}{r} 105 \\ 3\overline{)316} \\ -\ 3 \\ \hline 016 \\ -\ 0 \\ \hline 16 \\ -\ 15 \\ \hline 1 \end{array}$$

③

He can make 105 bags of oranges. ②
One orange will be left over. ②

b) (Use algebra)
Dan saved twice as much as Brett. Maria saved \$60 more than Brett.
If they saved \$600 altogether, how much did Maria save?

Let b : the amount Brett saved, in \$.
Then: Maria saved \$$(b+60)$,
Dan saved \$$2b$ ③

Total amount: $(2b)+(b+60)+(b)=600$
$4b+60=600$
$4b=540$
$b=135$ ③
(Brett saved \$135)

Maria: $b+60=135+60=195$ ②
Maria saved \$195 ②

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14) *(10 points)* Give an algebraic proof for the divisibility test for 8:
A number is divisible by 8 if and only if its last 3 digits form a number divisible by 8.

By PV, any whole # n can be written as: ②

$n = 1000a + b = \underbrace{8(125a)}_{\text{div. by } 8} + b$ ↳ test #: last 3 digits
① ① ①

By DL, $8 \mid n \iff 8 \mid b$ (i.e. last 3 digits form a number div. by 8)
② ① ① ①

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