

Instructions:

You must show ALL your work in ALL questions. You will be graded on your methods, not just your answers. Use only the space provided for each question. Any usage of calculators is prohibited during the exam.

You will have EXACTLY 60 minutes for the exam, which consists of problems numbered 1 – 14. Request a new copy of the exam if any of the problems are missing or hard to read.

1) (2 points each) Fill in the blanks using the correct terminology in their full form:

- a) Jane put 14 pens equally into 4 boxes. When she was done;
There were _____ pens in each box, a total of _____ pens distributed, and _____ left over.
 - b) After each step in long division, it's necessary to check that $0 \leq \text{remainder} < \text{_____}$.
 - c) The most important property used in proving $(a + b)(c + d) = ac + bc + ad + bd$ is:
_____.
 - d) There are _____ many prime numbers.
 - e) In the addition $107 + 235$, the _____ denomination is rebundled.
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2) (2 points each) Complete the following definitions: (Hint: The space provided is sufficient!)

- a) An *algorithm* is a _____.
 - b) An *equation* is a statement that _____.
 - c) A is *divisible* by k if $A = \text{_____}$.
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3) (2 points each) True or False:

- a) $5 + 13 \div 8$ is an algebraic expression. T F
 - b) 321 is a prime number. T F
 - c) All prime numbers are odd. T F
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4) (2 points each) Show how to calculate the following using Mental Math techniques that involve, either squares of numbers up to 20 or powers of 2 up to the tenth power or arithmetic identities including squares of sums/differences, differences of squares. Clearly, but briefly, show the intermediate steps, and state the properties/techniques used.

a) $16 \times 14 =$

b) $16 \times 32 =$

c) $49^2 =$

5) (2 points) Explain, in a single short sentence, ONLY the mistake made:

$$\begin{array}{r} 25 \\ + 89 \\ \hline 1014 \end{array}$$

6) (5 points) Circle the numbers below that divide 243,276.

2 3 4 5 8 9 10 11

7) (5 points) Compute 36×82 using the lattice method.

8) (5 points) Complete the following long division:

$$43 \overline{) 2594} \begin{array}{r} 5 \\ \hline \end{array}$$

9) (6 points) Find the prime factorization of 1495 using its factor tree (and primality tests).

10) (5 points) Illustrate the use of the distributive property in the multiplication algorithm of 42×13 using a labeled rectangular array, showing the values appearing in the algorithm.

11) (6 points) Give a correctly labeled picture proof that the sum of two odd numbers is even.

12) (8 points) Give an algebraic proof of the identity $(a + b)(a - b) = a^2 - b^2$.

13) (10 points each) Give full Teacher Solutions for the following problems.

a) (Use a diagram and long division)

A man has 316 oranges. He puts 3 oranges in each bag.

How many bags of oranges can he make? How many oranges will be left over?

b) (Use algebra)

Dan saved twice as much as Brett. Maria saved \$60 more than Brett.

If they saved \$600 altogether, how much did Maria save?

14) (10 points) Give an algebraic proof for the divisibility test for 8:
A number is divisible by 8 if and only if its last 3 digits form a number divisible by 8.