MTH 202.1 FS 2007 EXAM 3 **Name:** ANSWER KEY **PID:** ______

Instructions:

You must show ALL the work required in ALL questions. Use only the space provided for each question. Read the statements of the questions very carefully. You will be graded on your methods, not just your answers.

You need a pencil, an eraser, a reliable compass, and a straightedge for this exam. Any use of rulers, protractors, and calculators is prohibited during the exam.

You will have EXACTLY 50 minutes for the exam, which consists of problems numbered 1 – 13. Request a new copy of the exam if any of the problems are missing or hard to read.

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1) (7 points) State whether each statement is "always true" (**T**) or "not always true" (**F**).

a) (2, 3, 5) is a Pythagorean Triple. T F

b) $\cos 60° = \frac{1}{2}$. T F

c) The value of π is $\frac{22}{7}$. T F

d) $\sqrt{2}$ is an irrational number. T F

e) (14, 48, 50) is a Pythagorean Triple. T F

f) $\sqrt{a}+\sqrt{b}=\sqrt{a+b}$. T F

g) If two lines (with slopes m_1 and m_2) are perpendicular, then $m_1 = -m_2$. T F

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2) (6 points) Complete the following conversions.

a) 4.3 kg = 4300 g

b) 12 ml = 0.012 l

c) 0.76 dm^2 = 7600 mm^2

d) 8,500,000 cm^3 = 8.5 m^3

e) 37 l = 0.037 m^3

f) 920 cm^3 = 920 ml

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3) (7 points)

a) Fill in the blanks: "In a right triangle, the perpendicular sides are called the legs ①."

b) Write down the complete precise definition of a *sector*.

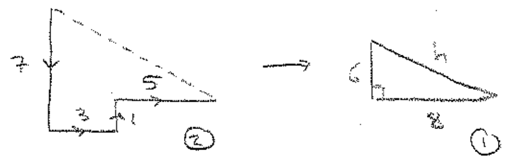
A sector of a disk is the portion of the disk that lies inside a central angle.
① ① ①

c) Write down the complete precise statement of the *Pythagorean Theorem*.

If a right triangle has legs of length a, b & hypotenuse of length c, then $a^2+b^2=c^2$
① ① ①

4) (6 points) You travel 7 miles due south, 3 miles due east, 1 mile due north, and then 5 more miles due east.

a) How far are you from your starting point?



Pyth. Triple : (6, 8, 10) $\Longrightarrow$ h = 10 miles (2)

b) On a map with scale 1 cm to 8 miles, this distance would be shown as $\frac{10}{8} = \frac{5}{4} = 1.25$ cm. (1)

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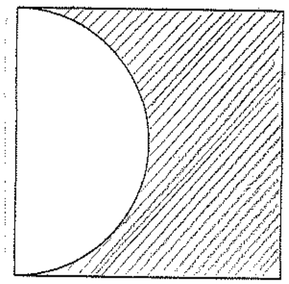
5) (6 points) The figure shows a square and a semicircle. Circle the area and the perimeter of the <u>shaded</u> part.



Area	$16(2-\pi)$	32π	$8(8-\pi)$	56π	$32(2-\pi)$
Perimeter	$4(6+\pi)$	28π	$4(8+\pi)$	32π	$8(3+\pi)$

$A(\text{region}) = \overbrace{A(\text{square}) - A(\tfrac{1}{2}\text{ circle})}^{1} = \overbrace{8^2 - \tfrac{1}{2}\cdot\pi\cdot 4^2}^{1} = \overbrace{64 - 8\pi = 8(8-\pi)}^{1}$

$P(\text{region}) = \underbrace{8+8+8+\tfrac{1}{2}C(\text{circle})}_{1} = \underbrace{24 + \tfrac{1}{2}\cdot 2\cdot\pi\cdot 4}_{1} = \underbrace{24 + 4\pi = 4(6+\pi)}_{1}$

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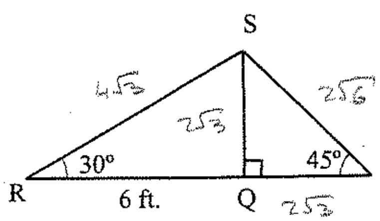
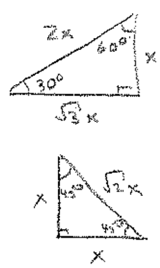
6) (8 points) In the figure, find the perimeter of ΔRST.





$|SQ| = \frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$ (2)

$|RS| = 2\cdot 2\sqrt{3} = 4\sqrt{3}$ (2)

$|QT| = 2\sqrt{3}$ (1)

$|ST| = 2\sqrt{3}\cdot\sqrt{2} = 2\sqrt{6}$ (2)

$P = 6 + 6\sqrt{3} + 2\sqrt{6}$ ft (1)

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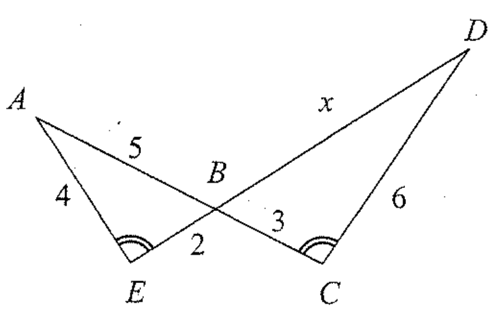
7) (12 points) In the figure, AC and DE are segments. [NOTICE: 2 DIFFERENT POSSIBLE METHODS!]



a) Give a 4-step similarity test:

S: $\frac{|BC|}{|AE|} = \frac{3}{2}$ (2)

A: $\angle AEB = \angle DCB$ (1) (given) (1)

S: $\frac{|DC|}{|AE|} = \frac{6}{4} = \frac{3}{2}$ (2)

Alternatively:

A: $\angle ABE = \angle DBC$ (1) (vert $\angle$s) (1)

A: $\angle AEB = \angle DCB$ (1) (given) (1)

A: $\angle BAE = \angle BDC$ (1) ($\angle$ sum of Δs) (1)

(equiangular) (1)

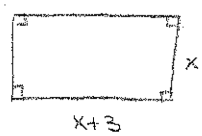
Δ ABE $\sim \Delta$ DBC (1) (ratio of 2 sides, incl. $\angle$ (1))

$x = \overbrace{5\cdot\tfrac{3}{2}}^{1} = \overbrace{\tfrac{15}{2} = 7.5}^{1}$ $\Longrightarrow$ b) $x =$ 7.5 cm.

$\underbrace{\text{Area s.f.} = \left(\tfrac{3}{2}\right)^2 = \tfrac{9}{4}}_{1}$ $\quad \frac{3.2\times 9}{4} = 7.2$ (1) $\Longrightarrow$ c) If Area(ΔABE) = 3.2 cm^2, then Area(ΔBCD) = 7.2 cm^2.

8) (8 points) Give a Teacher Solution including a sketch:

The length of a rectangle is 3 cm greater than its width. If the perimeter is 34 cm, find the width and the length.



1

Let x be the width of the rectangle in cm. 1

Then the length is $x+3$ cm. 1

Perimeter $= 2(w+\ell) = 2(x+x+3) = 2(2x+3)$ 1
1

So: $2(2x+3) = 34$
$2x+3 = 17$
$2x = 14$
width: $x = 7$ cm 1

length: $x+3 = 10$ cm 1

$\therefore$ the width is 7 cm & the length is 10 cm 1

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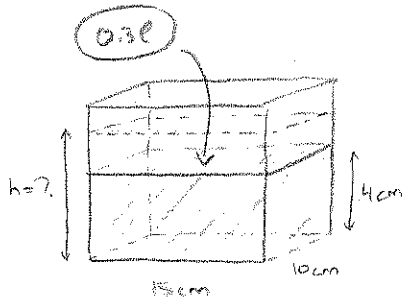
9) (8 points) Give a Teacher Solution including a sketch:

A rectangular container, 15 cm long and 10 cm wide, contains water to a depth of 4 cm.
What would be the new height of water, after a stone of volume 0.3 liters is submerged in the water?



2

$V(\text{Stone}) = 0.3 \text{ liters} = 0.3 \times 1000 \text{ cm}^3 = 300 \text{ cm}^3$ 1

$V(\text{Difference}) = V(\text{Stone}) = 300 \text{ cm}^3$ 1

So: $300 = 15 \times 10 \times (h-4)$ 2

$\frac{300}{150} = (h-4)$
$2 = h-4$
$h = 6$ cm 1

$\therefore$ The new height of water is 6 cm 1

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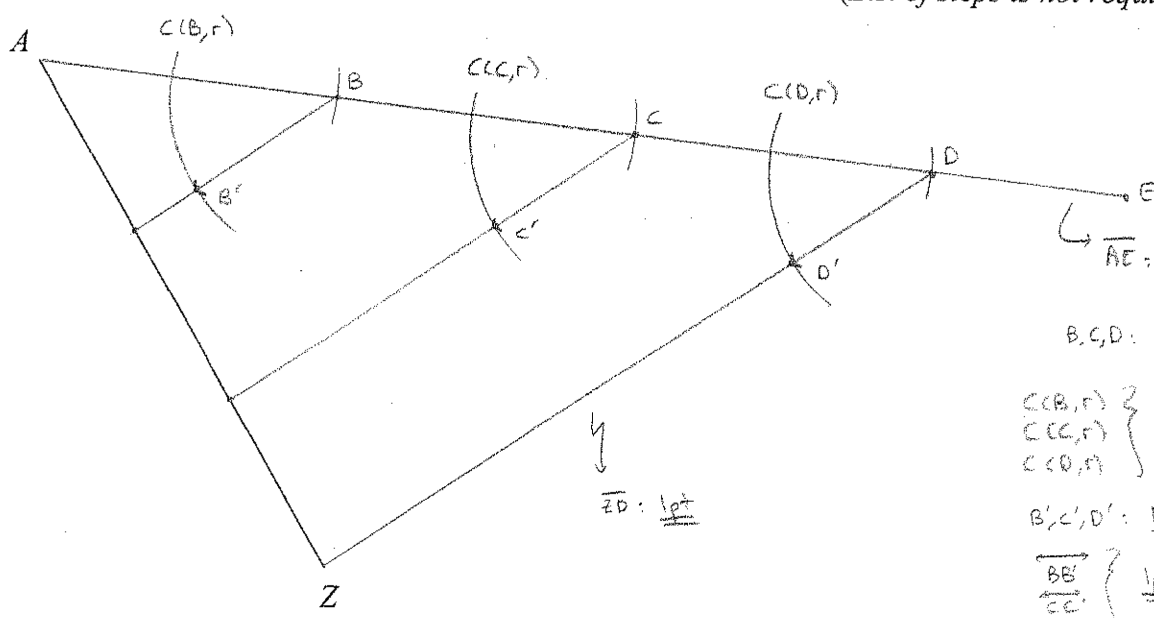
10) (8 points) Using your straightedge and compass, divide $\overline{AZ}$ into 3 equal parts.

(List of steps is not required!)



$\overline{AE}$: 1pt

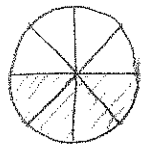
B, C, D : 2 pts

C(B,r), C(C,r), C(D,r) : 2 pts

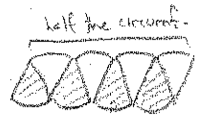
B', C', D' : 1pt

$\overline{BB'}$, $\overline{CC'}$, $\overline{DD'}$: 1pt

11) (8 points) Teacher Explanation: Draw pictures and give brief short-sentence explanations accompanying your pictures, to explain the "pizza method" demonstration of "the area formula of a circle".



① ⟶



① ⟶

①

Divide circle into sectors ①

Rearrange sectors into "almost" a parallelogram ①

Height is "almost" r, base is "almost" πr ①

$\Rightarrow$ Area $\approx (\pi r)\cdot r = \pi r^2$ ①

Repeatedly using thinner & thinner slices, Area gets closer & closer to πr^2

$\Rightarrow$ Area (Circle) $= \pi r^2$ ①

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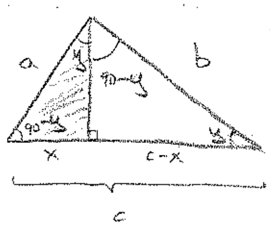
12) (8 points) Teacher Explanation: Draw pictures and give brief short-sentence explanations accompanying your pictures, to explain the "similar triangles" demonstration of "the Pythagorean Theorem".



Picture: ①

Labels: ②

Shaded Daughter ~ Mother $\Rightarrow \frac{a}{x} = \frac{c}{a} \Rightarrow a^2 = cx$ ②

Unshaded Daughter ~ Mother $\Rightarrow \frac{b}{c-x} = \frac{c}{b} \Rightarrow b^2 = c(c-x)$ ②

Add side-by-side ⟶ $a^2+b^2 = \cancel{cx} + (c^2 - \cancel{cx})$

$\Rightarrow a^2+b^2 = c^2$ ①

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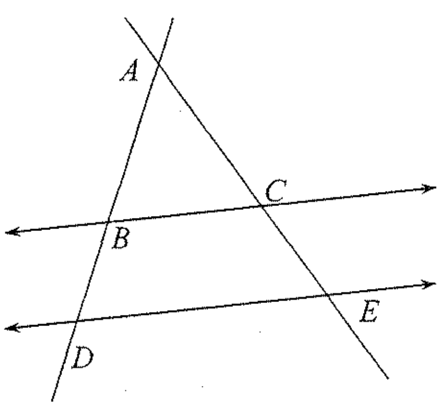
13) (8 points) In the picture $\overleftrightarrow{BC} \parallel \overleftrightarrow{DE}$. Prove that the two triangles are similar.



Given: $\overline{BC} \parallel \overline{DE}$ ①

To Prove: $\triangle ABC \sim \triangle ADE$ ①

Proof: $\angle BAC = \angle DAE$ (common) ②

$\angle ABC = \angle ADE$ (corr. ∠s, $\overline{BC} \parallel \overline{DE}$) ②

[$\angle ACB = \angle AED$ (corr ∠s, $\overline{BC} \parallel \overline{DE}$)] (optional)

∴ $\triangle ABC \sim \triangle ADE$ (equiangular) ②